EXCITATION OF WAVES OF INSTABILITY OF THE SECONDARY FLOW IN THE BOUNDARY LAYER ON A SWEPT WING

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For practically important cases of low-level background disturbances, the laminar-turbulent transition of the boundary layer is determined by the excitation and development of unstable perturbations [1, 2]. As a rule, two types of instability develop on high-aspect-ratio swept wings: Tollmin–Schlichting waves (in the middle part of the wing profile); instability of the secondary flow (on the fore and aft parts of the profile, where the boundary layer is essentially three-dimensional). Characteristics of the development of both types of instability were examined in detail in [3-6]. The excitation of Tollmin–Schlichting waves in three-dimensional boundary layers was examined in [7, 8]. However, no attention has been given to study of the excitation of instability in secondary flows. There is also no experimental data on the mechanisms of its generation.

Here, we theoretically analyze the excitation of waves of instability of the secondary flow on a swept wing in a boundary layer undergoing compression. The sources of generation are local flow discontinuities due to roughness or vibration of the surface, its heating or cooling, or the suction or injection of gas through the permeable surface.

1. We will examine flow in the laminar boundary layer on a swept wing. An external load which is harmonic over time and which is localized on scales on the order of the length of an instability wave is applied to the surface of the fore part of the wing profile in the region of intensive secondary flow. The intensity of the load is fairly low, and the perturbations are described by linear theory. The external load excites waves of instability in the secondary flow. It is necessary to determine the amplitudes of these waves.

We introduce a coordinate system with its origin at the center of the loading section, as shown in Fig. 1. The x axis is directed along the surface in the flow perpendicularly to the leading edge of the wing. The z axis is directed along the leading edge, while the y axis is directed along a normal to the wing surface. The coordinates x, y, and z are made dimensionless relative to the displacement thickness δ^*, calculated from the profile of the x component of the velocity of the main flow U(y). It is assumed that the Reynolds number $R_L = U_eL/\nu_e \gg 1$ (L is the chord of the wing, ν is the kinematic viscosity, and the subscript e denotes a quantity on the external boundary of the boundary layer). Since the length of the instability wave is commensurate with the thickness of the boundary layer δ, the generation is localized on the scale $\delta \ll L$ and we can ignore the effects of nonuniformity of the main flow with respect to x.

For the perturbations, we introduce the vector function

$$\mathbf{\Psi}(x, y, z, t) = \left(u, \frac{\partial u}{\partial y}, v, p, \theta, \frac{\partial \theta}{\partial y}, w, \frac{\partial w}{\partial y}, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial \theta}{\partial x}, \frac{\partial w}{\partial x}, \frac{\partial u}{\partial z}, \frac{\partial v}{\partial z}, \frac{\partial \theta}{\partial z}, \frac{\partial w}{\partial z}\right),$$

where u, v, and w are perturbations of the x, y, and z components of velocity, referred to U_e; p is the perturbation of pressure, made dimensionless with respect to $\rho_e U_e^2$ (ρ is density); θ is the temperature perturbation, referred to the temperature of the main flow T_e; t is the time in the units δ^*/U_e.

We will examine a perturbation of fixed frequency ω, which we will represent in the form $\mathbf{\Psi}(x, y, z, t) = \mathrm{Re}[\mathbf{A}(x, y, z)\exp(-i\omega t)]$ ($\mathbf{A}$ is the complex amplitude).

We linearize the Navier–Stokes equations, discarding the terms due to nonuniformity of the main flow along x. We then perform a Fourier transformation with respect to time. This gives us a system of equations for the vector function

$$\frac{\partial}{\partial y}\left(L_0 \frac{\partial \mathbf{A}}{\partial y}\right) + L_1 \frac{\partial \mathbf{A}}{\partial y} = H_1 \mathbf{A} + H_2 \frac{\partial \mathbf{A}}{\partial x} + H_3 \frac{\partial \mathbf{A}}{\partial z} \tag{1.1}$$

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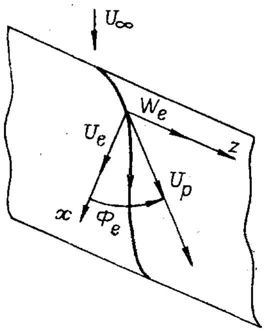

Fig. 1

(L_0, L_1, H_1, H_2, H_3 are matrices of the dimensions 16 × 16). Their elements depend on the main flow, the frequency ω, and $R = U_e\delta^*/\nu_e$. The external loading leads to inhomogeneous boundary conditions on the surface in the flow

$$(A_1, A_3, A_5, A_7) = (\varphi_1, \varphi_2, \varphi_3, \varphi_4),\ y = 0 \tag{1.2}$$

(φ_j are components of the vector function $\varphi(x, z)$ which describes the loading).

It is assumed that the perturbations decay upflow and in the direction of the span of the wing. The perturbations may increase downflow with finite growth exponents. We have the boundary conditions

$$|\mathbf{A}| \to 0,\ y \to \infty,\ x, z = \text{const};\ x \to -\infty,\ y, z = \text{const}; \\ z \to \pm\infty,\ y, x = \text{const}. \tag{1.3}$$

Within the framework of linear theory, problem (1.1)-(1.3) describes the excitation and development of perturbations in the vicinity of the section subjected to external loading.

2. We seek the solution of problem (1.1)-(1.3) in the form of an expansion in a biorthogonal system of eigenfunctions $\{\mathbf{A}_{\alpha\beta}(y), \mathbf{B}_{\alpha\beta}(y)\}$ determined by the direct and conjugate problems with homogeneous boundary conditions at y = 0 [8]

$$\frac{d}{dy}\left(L_0\frac{d\mathbf{A}_{\alpha\beta}}{dy}\right) + L_1\frac{d\mathbf{A}_{\alpha\beta}}{dy} = H_1\mathbf{A}_{\alpha\beta} + i\alpha H_2\mathbf{A}_{\alpha\beta} + i\beta H_3\mathbf{A}_{\alpha\beta}, \tag{2.1}$$
$$A_{\alpha\beta,1} = A_{\alpha\beta,3} = A_{\alpha\beta,5} = A_{\alpha\beta,7} = 0,\quad y = 0,\quad |\mathbf{A}_{\alpha\beta}| < \infty,\quad y \to \infty;$$

$$\frac{d}{dy}\left(L_0^*\frac{d\mathbf{B}_{\alpha\beta}}{dy}\right) - L_1^*\frac{d\mathbf{B}_{\alpha\beta}}{dy} = H_1^*\mathbf{B}_{\alpha\beta} - i\bar{\alpha}H_2^*\mathbf{B}_{\alpha\beta} - i\bar{\beta}H_3^*\mathbf{B}_{\alpha\beta}, \tag{2.2}$$
$$B_{\alpha\beta,2} = B_{\alpha\beta,4} = B_{\alpha\beta,6} = B_{\alpha\beta,8} = 0,\quad y = 0,\quad |\mathbf{B}_{\alpha\beta}| < \infty,\quad y \to \infty,$$

where * denotes the transposed matrix with complex-conjugate elements; the superimposed bar denotes complex conjugation.

The eigenfunctions $\mathbf{A}_{\alpha\beta}(y)$ describe the amplitude of a wave of the form $\mathbf{A}_{\alpha\beta}\exp(i\alpha x + i\beta z - i\omega t)$. Equations (2.1) are equivalent to the Less–Lin system of equations normally used in numerical analyses of the stability of compressible three-dimensional boundary layers [3]. The following conditions of orthogonality are satisfied

$$\langle H_2\mathbf{A}_{\alpha\beta}, \mathbf{B}_{\gamma\beta}\rangle = \Delta_{\alpha\gamma},\quad \langle H_2\ \mathbf{A}, \mathbf{B}\rangle = \int_0^\infty (H_2\mathbf{A}, \mathbf{B})\,dy,$$

$$(H_2\mathbf{A}, \mathbf{B}) = \sum_{j,k=1}^{16} H_2^{jk}A_k\bar{B}_j.$$

Here, $\Delta_{\alpha\gamma}$ is the Kronecker symbol if α or γ belong to a discrete spectrum; $\Delta_{\alpha\gamma} = \delta(\alpha - \gamma)$ is a delta function if α and γ belong to a continuous spectrum.

Since the boundary layer on a swept wing is independent of z, the coefficients in Eqs. (1.1) are also independent of z. We will expand the amplitude of the perturbation into a Fourier integral over the wave numbers β: $\mathbf{A}(x, y, z) = \int_{-\infty}^{\infty} \mathbf{Q}_\beta(x, y)\,e^{i\beta z}\,d\beta$.

In a certain section x = x_0 downflow of the loading section, the amplitude satisfies homogeneous boundary conditions at y = 0 and the condition to boundedness at $y \to \infty$. Thus, in the region $x > x_0$, $\mathbf{Q}_\beta$ can be expanded into eigenfunctions $\{\mathbf{A}_{\alpha\beta}, \mathbf{B}_{\alpha\beta}\}$:

$$\mathbf{Q}_\beta(x, y) = \sum_\alpha{}' \langle H_2\mathbf{Q}_\beta(x_0, y), \mathbf{B}_{\alpha\beta}\rangle \mathbf{A}_{\alpha\beta}(y) \exp[i\alpha(x - x_0)] \tag{2.3}$$

($\sum_\alpha{}'$ denotes summation over the discrete spectrum and integration over the continuous spectrum). The authors of [9] proved the completeness of the system of eigenfunctions for two-dimensional perturbations in a plane-parallel boundary layer of a compressible gas. This proof is easily generalized to the case of three-dimensional perturbations of the form Q(x, y) exp (iβz) (β is a real quantity) for the boundary layer on a swept wing.

Due to the linearity of the problem, we can analyze the excitation of each mode separately. We will restrict ourselves to examining waves of instability of the secondary flow. Characteristics of these wave solutions were studied in detail in [3-6]. For completeness of exposition, we will enumerate their main properties. Instability of the secondary flow is due to the point of inflection in the profile of the velocity component normal to the external streamline. The instability is inviscid in character. Its maximum increments are realized at frequencies close to zero. The vector of group velocity is close in direction to the velocity of the external flow and makes an angle ≈90° with the wave vector, i.e., the instability wave has a structure which is periodic with respect to the z axis, and it intensifies exponentially downflow. The determining parameters for secondary-flow instability are the Mach number M_e, the angle Φ_e between the streamline of the external flow and the x axis (see Fig. 1), the pressure-gradient parameter $\Lambda_e = 2\xi dU_e/d\xi/U_e$, $\xi = \int_0^x \rho_e U_e \mu_e \, dx$, the frequency ω, the z component of the wave vector, and R. We will use the subscript CF to denote characteristics of secondary-flow instability.

The coefficients in expansion (2.3) are determined as was done in [10], by analysis of the excitation of Tollmin–Schlichting waves by local loading in a plane-parallel boundary layer. We will examine the regime of subcritical frequencies. In this case the perturbation $\mathbf{Q}_\beta(x, y) \to 0$ at $x \to \pm\infty$, $y = \text{const}$. Expanding $\mathbf{Q}_\beta$ and $\boldsymbol{\varphi}$ into Fourier integrals, we have

$$\mathbf{Q}_\beta = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathbf{A}_v(y)\, e^{i\alpha_v x}\, d\alpha_v, \quad \boldsymbol{\varphi} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\beta e^{i\beta z}\int_{-\infty}^{\infty} \mathbf{f}(\alpha_v, \beta)\, e^{i\alpha_v x}\, d\alpha_v, \tag{2.4}$$
$$\frac{d}{dy}\left(L_0\frac{d\mathbf{A}_v}{dy}\right) + L_1\frac{d\mathbf{A}_v}{dy} = H_1\mathbf{A}_v + i\alpha_v H_2\mathbf{A}_v,$$
$$(A_{v,1}, A_{v,3}, A_{v,5}, A_{v,7}) = (f_1, f_2, f_3, f_4),\ y = 0,\ |\mathbf{A}_v| \to 0,\ y \to \infty.$$

The Fourier component $\mathbf{A}_v$ describes a perturbation introduced into the boundary layer by a harmonic external load with the wave numbers α_v, β. The following relation is satisfied

$$\langle H_2\mathbf{A}_v, \mathbf{B}_{\alpha\beta}\rangle i(\alpha_v - \alpha) + (\mathbf{A}_v, \mathbf{B}_{\alpha\beta})_{y=0} = 0. \tag{2.5}$$

It follows from (2.4) and (2.5) that the coefficient in front of the eigenfunction of the instability wave is

$$\langle H_2\mathbf{Q}_\beta(x_0, y), \mathbf{B}_{CF,\beta}\rangle = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{(\mathbf{A}_v, \mathbf{B}_{CF,\beta})_{y=0}}{(\alpha_v - \alpha)} \exp(i\alpha_v x_0)\, d\alpha_v. \tag{2.6}$$

To calculate the integral in (2.6), we choose the contour of integration in the complex plane α_v, as shown in Fig. 2. Making the radius r approach infinity and considering that $x_0 > 0$, we find

$$\langle H_2\mathbf{Q}_\beta, \mathbf{B}_{CF,\beta}\rangle = -(\mathbf{A}_v, \mathbf{B}_{CF,\beta})_{y=0} \exp(i\alpha_{CF} x_0). \tag{2.7}$$

It is not hard to show that

$$S \equiv -(\mathbf{A}_v, \mathbf{B}_{CF,\beta})_{y=0} = -\sum_{j=1}^{4}[f_j(\alpha_{CF}, \beta)\overline{B}_{CF,\beta,2j-1}]_{y=0}, \tag{2.8}$$

where $\mathbf{f}(\alpha_{CF}, \beta)$ is a harmonic of the external load which is resonant with the instability wave.

It follows from (2.7) and (2.8) that the amplitude of the excited mode of secondary flow $\mathbf{Q}_{CF,\beta}$ with the wave number β and the eigenvalue $\alpha_{CF} = \alpha_{CF}(\beta, \omega, R)$ has the form

$$\mathbf{Q}_{CF,\beta} = \frac{S}{\langle H_2 \mathbf{A}_{CF,\beta}, \mathbf{B}_{CF,\beta} \rangle} \mathbf{A}_{CF,\beta} \exp(i\alpha_{CF} x). \tag{2.9}$$

Introduction of the multiplier $\langle H_2 \mathbf{A}_{CF,\beta}, \mathbf{B}_{CF,\beta} \rangle^{-1}$ makes Eq. (2.9) invariant relative to the choice for normalization of the eigenfunctions.

In the transition from subcritical to supercritical eigenfrequencies, the value of α_{CF} intersects the real axis in the direction of the dashed arrow in Fig. 2. In this case, the rule for circumvention of the pole $\alpha_V = \alpha_{CF}$ is chosen in accordance with the postulate in [11] requiring continuity of the solution in the transition past the critical frequency. The contour of integration passes below the pole, as shown by the dashed lines in Fig. 2, and there is no change in the result (2.9). It should be noted that the solution of problem (1.1)-(1.3) is not unique for supercritical frequencies. In this case, the eigenfunction $\mathbf{A}_{CF,\beta} \exp[i\alpha_{CF} x]$, with an arbitrary coefficient, can be added to the solution. To make the solution unique, it is necessary to examine the problem with a load which is harmonic with respect to time. This load is applied at the initial moment t = 0. We subsequently proceed to the limit $t \to \infty$, as was done for Tollmin–Schlichting waves in [12]. Another method is to use the postulate in [11].

3. As was noted above, secondary-flow instability of zero frequency has a growth increment which is close to maximal. Such steady-state instabilities may be excited on irregularities on the surface in the flow. Let the form of an irregularity be $y(x, z) = a\varphi(x, z)$, $\varphi(x, z) = O(1)$ (a is the characteristic height of the irregularity, referred to δ^*). It is assumed that the roughness lies at the bottom of the viscous sublayer and results in inhomogeneous boundary conditions $A_1 = -aU'_w \varphi(x, z)$, $A_3 = A_5 = 0$, $A_7 = -aW'_w \varphi(x, z)$, $y = 0$, where U'_W, W'_W are derivatives of the x and z components of the velocity of the main flow with respect to y at y = 0.

Using Eq. (2.9), we obtain the following for the maximum modulus of the perturbation of the x component of velocity in the excited secondary-flow instability

$$q_{CF}(x, z) = a \int_{-\infty}^{\infty} G_r(\alpha_{CF}, \beta) f(\alpha_{CF}, \beta) \exp(i\alpha_{CF} x + i\beta z) \, d\beta, \tag{3.1}$$

$$q_{CF} = \max_y |u_{CF}(x, y, z)|,$$

$$G_r = \frac{(U'_w \overline{B}_{CF,\beta,1} + W'_w \overline{B}_{CF,\beta,7})_{y=0}}{\langle H_2 \mathbf{A}_{CF,\beta}, \mathbf{B}_{CF,\beta} \rangle} q_m(\alpha_{CF}, \beta).$$

Here, $f(\alpha_{CF}, \beta)$ is a resonance harmonic of the form of the irregularity φ ; G_r is the generation coefficient, equal to the initial amplitude of an instability wave with the wave number β excited on an irregularity with the resonance-harmonic amplitude $af(\alpha_{CF}, \beta) = 1$; $q_m = \max_y |A_{CF,\beta,1}|$ is the maximum of the modulus of the first component of the eigenvector function $\mathbf{A}_{CF,\beta}(y)$. The generation coefficient G_r is independent of the form of the roughness and the normalization of the eigenfunctions and is a universal characteristic of the efficiency of excitation of instability waves.

Let the external load take the form of steady heating (or cooling) of a section of the surface in the flow. Then $A_1 = A_3 = A_7 = 0$, $A_5 = \theta_0 \varphi(x, z)$, $y = 0$ (θ_0 is the characteristic heating temperature, referred to T_e). The amplitude of the excited instability wave is determined by Eq. (3.1) with the replacement of a by θ_0 and G_r by the generation coefficient G_T:

$$G_T = -\frac{\overline{B}_{CF,\beta,5}(0)}{\langle H_2 \mathbf{A}_{CF,\beta}, \mathbf{B}_{CF,\beta} \rangle} q_m(\alpha_{CF}, \beta). \tag{3.2}$$

If the external load is a weak suction (or injection) of gas through the permeable surface, then

$$A_1 = A_5 = A_7 = 0, \quad A_3 = v_0 \varphi(x, z), \quad y = 0, \tag{3.3}$$

$$G_s = -\frac{\overline{B}_{CF,\beta,3}(0)}{\langle H_2 \mathbf{A}_{CF,\beta}, \mathbf{B}_{CF,\beta} \rangle} q_m(\alpha_{CF}, \beta)$$

(v_0 is the characteristic rate of suction or injection, referred to U_e).

We can use Eqs. (3.1)-(3.3) to calculate the generation coefficients for subsonic flow velocities. Flow in the boundary layer was calculated in a local similarity approximation. The

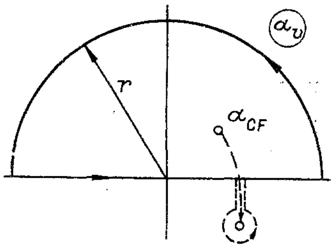

Fig. 2

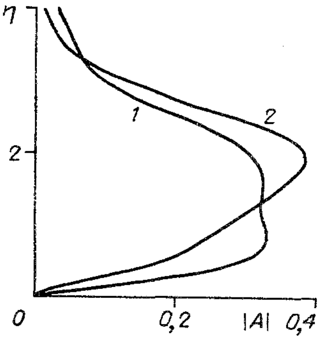

Fig. 3

TABLE 1

Λ_e	R	$\beta\cdot 10$	$\mathrm{Re}\,(\alpha_{CF})\cdot 10$	$\mathrm{Im}\,(\alpha_{CF})\cdot 10^3$	$\lvert G_r\rvert\cdot 10^3$	$\lvert G_T\rvert\cdot 10^3$	$\lvert G_s\rvert\cdot 10$
0,1	514,2	3,44	—3,95	0,11	5,64	2,07	3,61
—	771,3	3,61	—4,14	—2,37	4,43	1,42	3,72
—	1028,5	3,67	—4,21	—3,76	3,90	1,13	3,83
—	1285,6	3,69	—4,24	—4,66	3,60	0,97	3,93
0,5	128,6	2,90	—3,05	2,14	13,7	5,92	2,89
—	514,2	3,12	—3,27	—14,25	7,16	1,93	3,31
—	899,9	3,10	—3,26	—17,09	6,28	1,39	3,55
—	1285,6	3,12	—3,29	—18,27	5,80	1,14	3,68
0,9	96,4	2,66	—2,66	0,69	14,2	6,39	2,55
—	514,2	2,77	—2,75	—18,93	7,14	1,83	3,08
—	899,9	2,78	—2,77	—21,31	6,29	1,34	3,28
—	1285,6	2,81	—2,81	—22,33	5,92	1,14	3,46

eigenvalues and eigenfunctions α_{CF}, $\mathbf{A}_{CF,\beta}$, $\mathbf{B}_{CF,\beta}$ were calculated by the orthogonalization method with the use of the application package in [13]. Viscosity was determined from the Sutherland formula, the Prandtl number was 0.72, the stagnation temperature was 310 K, and the adiabatic exponent was 1.41. We examined the regimes: $M_e = 0.5$, $\Phi_e = 50°$, $\Lambda_e = 0.1$-0.9. The Reynolds number was varied from the value associated with loss of stability to 1300. The wave numbers β correspond roughly to the maximum growth increments. The results of the calculations are shown in Table 1.

Figure 3 shows distributions $|A_{CF,\beta,1}(\eta)|$ and $|A_{CF,\beta,7}(\eta)|$ (curves 1 and 2) which are typical of secondary-flow instability waves and correspond to the moduli of the perturbations of the x and z velocity components ;$\eta = \int\limits_0^y \rho/\rho_e\,dy$ is the Less–Dorodnitsyn variable. The eigenfunction was normalized with the condition $A_{Cf,\beta,2}(0) = 1$. Calculations were performed for the parameters of the main flow $\Lambda_e = 0.9$, $M_e = 0.5$, $\Phi_e = 50°$, $R = 1285.6$ and the instability-wave parameters $\omega = 0$, $\alpha_{CF} = -0.281 - i0.0223$. Analysis of the vector field shows that the instability wave is a system of longitudinal vortices localized within the boundary layer and oriented roughly in the direction of the velocity vector on the external boundary of the boundary layer.

To evaluate the efficiency of the above-examined excitation mechanism, we choose an externally loaded section in the form of a rectangle with the sides $x - l_x = \pi/|\alpha_{CF}|$, $z - l_z = \pi/|\beta|$. Let the loading be constant over the entire section, $\varphi(x, z) = 1$, $|x| < l_x/2$, $|z| < l_z/2$; $\varphi = 0$, $|x| > l_x/2$, $|z| > l_z/2$. Then $f(\alpha_{CF}, \beta) = 2/|\pi\alpha_{CF}\beta|$, and the initial amplitude of the instability wave with the wave number β will have the form

$$Q_{CF,0} = \frac{2a}{\pi\,|\alpha_{CF}\beta|}\,|G_r|,\quad \frac{2\theta_0}{\pi\,|\alpha_{CF}\beta|}\,|G_T|,\quad \frac{2v_0}{\pi\,|\alpha_{CF}\beta|}\,|G_s|.$$

It follows from the table that $|\alpha_{CF}\beta| \simeq 10^{-1}$, $|G_r| \simeq 10^{-2}$, $|G_T| \simeq 10^{-3}$, $|G_s| \simeq 3\cdot 10^{-1}$. For the initial amplitude $Q_{CF,0} = 0.1\%$, we obtain $a \simeq 10^{-2}$, $\theta_0 \simeq 10^{-1}$, $v_0 \simeq 10^{-4}$. Thus, to excite a secondary-flow instability wave with an initial amplitude of 0.1%, it is sufficient to have microroughness of the height $\cong 10^{-2}\delta^*$ or weak suction/injection at the rate $10^{-4}\,U_e$. Such excitations are almost unavoidable. Sources of these excitations may be roughness of the skin of the wing, slits, joints, etc. As in the case of the generation of Tollmin-Schlichting waves [10], local heating of the surface is less effective.

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